

# EXTENSIONS OF THE STANDARD MODEL

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This talk begins with a brief general introduction to the extensions of the Standard Model, reviewing the ideology of effective field theories and its practical implications. The central part deals with candidate extensions near the Fermi scale, focusing on some phenomenological aspects of the Minimal Supersymmetric Standard Model. The final part discusses some possible low-energy implications of further extensions near the Planck scale, namely superstring theories.

## 1 Preamble (some facts and some ideology)

It is quite obvious that the Standard Model (SM) **must** be extended. Among the ‘hard’ arguments supporting the previous statement, the strongest one is the fact that the SM does not include a quantum theory of gravitational interactions. Immediately after, one can mention the fact that some of the SM couplings are not asymptotically free, making it almost surely inconsistent as a formal Quantum Field Theory. One can add to the above the usual ‘soft’ argument that the SM has about 20 arbitrary parameters, which may seem too many for a fundamental theory.

Whilst this does not give us direct information on the form of the required SM extensions, it brings along an important conceptual implication: the SM should be seen as an **effective field theory**, valid up to some physical cut-off scale  $\Lambda$ . The basic rule of the game<sup>1</sup> is to write down the most general local Lagrangian compatible with the SM symmetries [i.e. the  $SU(3) \times SU(2) \times U(1)$  gauge symmetry and the Poincaré symmetry], scaling all dimensionful couplings by appropriate powers of  $\Lambda$ . The resulting dimensionless coefficients are then to be interpreted as parameters, which can be either fitted to experimental data or (if one is able to do so) theoretically determined from the fundamental theory replacing the SM at the scale  $\Lambda$ . Very schematically (and omitting all coefficients and indices, as well as many theoretical subtleties, such as the problems in regularizing chiral gauge theories):

$$\begin{aligned} \mathcal{L}_{eff} = & \Lambda^4 + \Lambda^2 \Phi^2 \\ & + (D\Phi)^2 + \bar{\Psi} \not{D} \Psi + F^2 + \bar{\Psi} \Psi \Phi + \Phi^4 \\ & + \frac{\bar{\Psi} \Psi \Phi \Phi}{\Lambda} + \frac{\bar{\Psi} \Psi \bar{\Psi} \Psi}{\Lambda^2} + \dots, \end{aligned} \quad (1)$$

where  $\Psi$  stands for the generic quark or lepton field,  $\Phi$  for the SM Higgs field, and  $F$  for the field strength of

the SM gauge fields. The first line of eq. (1) contains two operators carrying positive powers of  $\Lambda$ , a cosmological constant term proportional to  $\Lambda^4$  and a scalar mass term proportional to  $\Lambda^2$ . Barring for the moment the discussion of the cosmological constant term, which becomes relevant only when the model is coupled to gravity, it is important to observe that **no** quantum SM symmetry is recovered by setting to zero the coefficient of the scalar mass term. On the contrary, the SM gauge invariance forbids fermion mass terms of the form  $\Lambda \bar{\Psi} \Psi$ . The second line of eq. (1) contains operators with no power-like dependence on  $\Lambda$ , but only a milder, logarithmic dependence, due to infrared renormalization effects. The operators of dimension  $d \leq 4$  exhibit two remarkable properties: all those allowed by the symmetries are actually present in the SM; both baryon number and the individual lepton numbers are automatically conserved. The third line of eq. (1) is indeed the starting point of an expansion in inverse powers of  $\Lambda$ , containing infinitely many terms. For energies and field VEVs much smaller than  $\Lambda$ , the effects of these operators are suppressed, and the physically most interesting ones are those that violate some accidental symmetries of the  $d \leq 4$  operators. For example, a  $d = 5$  operator of the form  $\bar{\Psi} \Psi \Phi \Phi$  can generate a lepton-number-violating Majorana neutrino mass of order  $G_F^{-1}/\Lambda$  (where  $G_F^{-1/2} \simeq 300$  GeV is the Fermi scale), as in the see-saw mechanism; some of the  $d = 6$  four-fermion operators can be associated with flavour-changing neutral currents (FCNC) or with baryon- and lepton-number-violating processes such as proton decay.

At this point, the question that naturally emerges is the following: where is the cut-off scale  $\Lambda$ , at which the expansion of eq. (1) loses validity and the SM must be replaced by a more fundamental theory? Two extreme but plausible answers can be given:

- (I)  $\Lambda$  is not much below the Planck scale,  $M_P \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$  GeV, as roughly suggested

by the measured strength of the fundamental interactions, including the gravitational ones.

- (II)  $\Lambda$  is not much above the Fermi scale, as suggested by the idea that new physics must be associated with the mechanism of electroweak symmetry breaking.

In the absence of an explicit realization at a fundamental level, each of the above answers can be heavily criticized. The criticism of (I) has to do with the existence of the ‘quadratically divergent’ scalar mass operator, which becomes more and more ‘unnatural’ as  $\Lambda$  increases above the electroweak scale<sup>2</sup>. On general theoretical grounds, we would expect for such operator a coefficient of order 1, but experimentally we need a strongly suppressed coefficient, of order  $G_F^{-1}/\Lambda^2$ . However, after taking into account quantum corrections, this coefficient can be conceptually decomposed into the sum of two separate contributions, controlled by the physics below and above the cut-off scale, respectively. Answer (I) would then require a subtle (malicious?) conspiracy between low-energy and high-energy physics, ensuring the desired fine-tuning. The criticism of (II) has to do instead with the  $d > 4$  operators: in order to sufficiently suppress the coefficients of the dangerous operators associated with proton decay, FCNC, etc., the new physics at the cut-off scale  $\Lambda$  must have quite non-trivial properties!

At the moment, answer (I) is not very popular in the physics community, since we do not have the slightest idea on how the required conspiracy could possibly work at the fundamental level. Conceptually, such a possibility can be theoretically tested in an ultraviolet-finite Theory of Everything: as daring as it may sound, with the advent and the continuing development of string theories, we may not be very far from the implementation of the first quantitative tests. More concretely, such a possibility can be experimentally tested in the near future, via the search for the Higgs boson at LEP, at the Tevatron and at the LHC. A clear picture of the implications of (I) is given in figure 1, which shows, for various possible choices of  $\Lambda$  in the SM, the values of the top quark and Higgs boson masses allowed by the following two requirements<sup>3</sup>:

- The SM effective potential should not develop, besides the minimum corresponding to the experimental value of the electroweak scale, other minima with lower energy and much larger value of the Higgs field. In first approximation, this amounts to requiring the SM effective Higgs self-coupling,  $\lambda(Q)$ , not to become negative at any scale  $Q < \Lambda$ : for a given value of the top quark mass  $M_t$ , this sets a lower bound on the SM Higgs mass  $m_H$ .
- The SM effective Higgs self-coupling should not develop a Landau pole at scales smaller than  $\Lambda$ : for a given value of  $M_t$ , this sets an upper bound on

$m_H$ . Such constraint has a meaning which goes beyond perturbation theory, as suggested by the infrared structure of the SM renormalization group equation for  $\lambda(Q)$  and confirmed by explicit lattice computations<sup>4</sup>.

Figure 1 includes some recent refinements<sup>5</sup> of the original analysis, such as two-loop renormalization group equations, optimal scale choice, finite corrections to the pole top and Higgs masses, etc. For very large cut-off scales,  $\Lambda = 10^{16}$ – $10^{19}$  GeV, the results are quite stable and can be summarized as follows: for a top quark mass close to 180 GeV, as measured at the Tevatron collider<sup>6</sup>, the only allowed range for the SM Higgs mass is 130 GeV  $< m_H < 200$  GeV. This means that, even in the absence of a direct discovery of new physics beyond the SM, answer (I) could be falsified by LEP, the Tevatron and the LHC in two possible ways: either by discovering a SM-like Higgs boson lighter than 130 GeV, or by excluding a SM-like Higgs boson in the 130–200 GeV range!

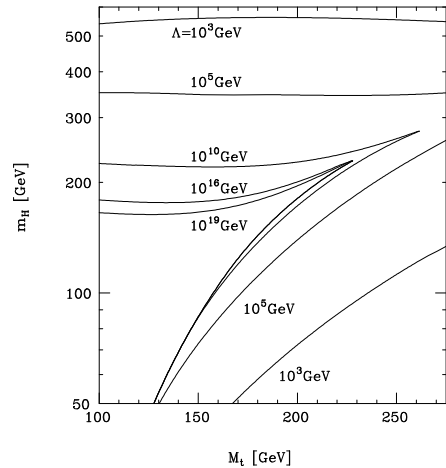


Figure 1: Bounds in the  $(M_t, m_H)$  plane, for various choices of  $\Lambda$ .

Answer (II), instead, gives rise to a well-known conceptual bifurcation:

- (IIa) In the description of electroweak symmetry breaking, the elementary SM Higgs scalar is replaced by some fermion condensate, induced by a new strong interaction near the Fermi scale. This includes old and more recent variants of the so-called **technicolor** models<sup>7</sup> (‘extended’, ‘walking’, ‘non-commuting’, ...). The stringent phenomenological constraints on technicolor models coming from electroweak precision data will be mentioned later. On the theoretical side, technicolor remains quite an appealing idea, still waiting for a satisfactory and calculable model. The lack of substantial theoretical progress in this field, however, may be due to the technical difficulties of dealing with intrinsically non-perturbative phenomena. This should not and certainly will not prevent the experimentalists from

keeping an open mind when looking for possible signals of new physics.

(IIb) The SM is embedded in a model with softly broken global supersymmetry, and supersymmetry-breaking mass splittings between the SM particles and their superpartners are of the order of the electroweak scale. This approach, generically denoted as **low-energy supersymmetry**<sup>8</sup>, ensures the absence of field-dependent quadratic divergences, and makes it ‘technically’ natural that there exists scalar masses much smaller than the cut-off scale. Moreover, a minimal and calculable model is naturally singled out, the so-called Minimal Supersymmetric Standard Model (MSSM).

## 2 Extensions near the Fermi scale (mainly MSSM phenomenology)

This section reviews some phenomenological aspects of SM extensions near the Fermi scale. Reflecting the content of the parallel sessions and the personal taste of the speaker, most of it will deal with the MSSM and its variants.

In order to set the framework for the following discussion, it is useful to recall the defining assumptions of the MSSM. The field content is organized in gauge and matter multiplets of  $N = 1$  supersymmetry. The gauge group is  $G = SU(3)_C \times SU(2)_L \times U(1)_Y$ , and the matter content corresponds to three generations of quarks and leptons, as in the SM, plus two complex Higgs doublets, one more than in the SM. To enforce baryon- and lepton-number conservation in  $d = 4$  operators, one imposes a discrete  $R$ -parity:  $R = +1$  for all ordinary particles (quarks, leptons, gauge and Higgs bosons),  $R = -1$  for their superpartners (spin-0 squarks and sleptons, spin-1/2 gauginos and higgsinos). A globally supersymmetric Lagrangian  $\mathcal{L}_{\text{SUSY}}$  is then fully determined by the superpotential (in standard notation):

$$f = h^U Q U^c H_2 + h^D Q D^c H_1 + h^E L E^c H_1 + \mu H_1 H_2. \quad (2)$$

To proceed towards a realistic model, one has to introduce supersymmetry breaking. In the MSSM, supersymmetry breaking is *parametrized* by a collection of *soft* terms,  $\mathcal{L}_{\text{soft}}$ , which preserve the good ultraviolet properties of global supersymmetry.  $\mathcal{L}_{\text{soft}}$  contains mass terms for scalar fields and gauginos, as well as a restricted set of scalar interaction terms

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= \sum_i \tilde{m}_i^2 |\varphi_i|^2 + \frac{1}{2} \sum_A M_A \bar{\lambda}_A \lambda_A \\ &+ (h^U A^U Q U^c H_2 + h^D A^D Q D^c H_1 \\ &+ h^E A^E L E^c H_1 + m_3^2 H_1 H_2 + \text{h.c.}), \quad (3) \end{aligned}$$

where  $\varphi_i$  ( $i = H_1, H_2, Q, U^c, D^c, L, E^c$ ) denotes the generic spin-0 field, and  $\lambda_A$  ( $A = 1, 2, 3$ ) the generic gaugino field. Observe that, since  $A^U, A^D$  and  $A^E$  are matrices in generation space,  $\mathcal{L}_{\text{soft}}$  contains in principle a huge number of free parameters. Moreover, for generic values of these parameters one encounters phenomenological problems with FCNC, CP violation, charge- and colour-breaking vacua. All the above problems can be solved at once if one assumes that the running mass parameters in  $\mathcal{L}_{\text{soft}}$ , defined at the one-loop level and in a mass-independent renormalization scheme, can be parametrized, at a cut-off scale  $\Lambda$  close to  $M_P$ , by a universal gaugino mass  $m_{1/2}$ , a universal scalar mass  $m_0$ , and a universal trilinear scalar coupling  $A$ , whereas  $m_3^2 \equiv -B\mu$  remains in general an independent parameter.

### 2.1 MSSM (and alternatives) vs. electroweak precision data

The theoretical interpretation of electroweak precision data, in the framework of the SM and of its candidate extensions (including the MSSM), has been the subject of several talks in the parallel<sup>9</sup> and plenary<sup>10,11</sup> sessions.

Universal effects, occurring via the vector-boson self-energies, and parametrized in terms of convenient variables<sup>12</sup> such as  $(S, T, U)$  or  $(\epsilon_1, \epsilon_2, \epsilon_3)$ , have already been discussed many times at this and previous conferences, and the results can be summarized as follows:

- The SM fits excellently all the data (with the value of the strong coupling constant extracted from the hadronic  $Z$  and  $\tau$  branching ratios slightly higher than, but still compatible with, the one extracted from deep-inelastic scattering).
- The MSSM gives at least as good a fit as the SM, thanks to the fast decoupling properties of the virtual effects of supersymmetric particles, as long as their mass is increased above the  $m_Z/2$  threshold.
- Naive versions of technicolor and extended technicolor models are ruled out (whereas some ‘walking’ technicolor models may still work).

A point that has attracted increasing attention in the months before this Conference is the fact that, in some extensions of the SM, non-universal effects on the  $Zb\bar{b}$  vertex are also possible, which can modify appreciably the SM prediction for  $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ . In the MSSM, one can have<sup>13</sup> extra positive contributions to  $R_b$  from loops involving stop squarks and charginos or bottom quarks and neutral Higgs bosons, extra negative contributions to  $R_b$  from loops involving the top quark and the charged Higgs boson. In the technicolor framework, one has<sup>14</sup> extra negative contributions to  $R_b$  in

‘walking’ technicolor models, whereas contributions can be of either sign in ‘non-commuting’ technicolor models.

The experimental data available before this Conference<sup>15</sup> suggested<sup>16</sup> that an improved fit to  $\alpha_S$  and  $R_b$  could be obtained in the MSSM in the case of light stops and charginos (with generic  $\tan\beta$ ) and/or light  $A^0$  (with  $\tan\beta \sim m_t/m_b$ ). For the effect to be numerically significant, the non-standard particles in the loops should not be much heavier than  $m_Z/2$ , otherwise fast decoupling would take place and the effect rapidly vanish. A quantitative estimate of this effect is given<sup>17</sup> in figure 2, which includes, besides the standard  $(t, W^\pm)$  loop, also the  $(t, H^\pm)$  loop and the  $(\tilde{t}, \tilde{\chi}^\pm)$  loops, in the simplified case of light  $\tilde{t}_R$  and  $\tilde{H}^\pm$ .

After the new data presented at this Conference<sup>11</sup>, the picture appears more confused. Now both  $R_b$  and the analogous ratio  $R_c$  have been measured to better accuracy and with different methods. The theoretical SM prediction [for  $M_t = 180 \pm 12$  GeV,  $m_H = 65\text{--}1000$  GeV,  $\alpha_S(m_Z) = 0.125 \pm 0.007$  and  $\alpha^{-1}(m_Z) = 128.90 \pm 0.09$ ] and the averaged experimental determinations are:

	EXP.	TH.(SM)
$R_b$	$0.2219 \pm 0.0017$	$0.2156 \pm 0.0005$
$R_c$	$0.1540 \pm 0.0074$	$0.1724 \pm 0.0003$

We then have an excess in  $R_b$  at about the  $3.5\sigma$  level and a defect in  $R_c$  at about the  $2.5\sigma$  level. Taking into account the measured value of the total hadronic width,

$$\Gamma_{h,exp} = (1744.8 \pm 3.0) \text{ MeV}, \quad (4)$$

which comfortably agrees with the SM prediction,

$$\Gamma_{h,SM} = (1745.7 \pm 6.0) \text{ MeV}, \quad (5)$$

one finds the following discrepancies:  $\delta\Gamma_b = 11 \pm 3$  MeV,  $\delta\Gamma_c = -32 \pm 13$  MeV,  $\delta(\Gamma_b + \Gamma_c) = -21 \pm 12$  MeV. Notice that the discrepancy in  $\Gamma_b + \Gamma_c$  is much larger than the error on  $\Gamma_h$ , and in sign and magnitude cannot support any longer any intriguing connection between the experimental effects on  $\alpha_S$  and  $R_b$ . Also, fitting the data within the MSSM now becomes impossible: for stops, charginos and  $A^0$  all around 50 GeV, and  $\tan\beta \sim m_t/m_b$ , the MSSM could marginally reproduce the observed value of  $R_b$ , but the improvement in the fit to  $R_c$  with respect to the SM would be negligible.

One could then fix (somewhat arbitrarily)  $R_c$  to its SM value. In this case, the fit to the experimental data would give  $R_b = 0.2205 \pm 0.0016$ , roughly  $3\sigma$  in excess of the SM prediction. Still, as can be appreciated from

figure 2, the discrepancy would be large enough that, barring very special regions of the parameter space, which may be already ruled out by indirect constraints or soon ruled out by the forthcoming LEP run at  $\sqrt{s} = 130\text{--}140$  GeV, the MSSM can provide only a modest improvement in the quality of the fit.

## 2.2 MSSM and the decay $b \rightarrow s\gamma$

As discussed in the parallel sessions<sup>18,19</sup>, the recent experimental observation of radiative  $B$  decays<sup>20</sup> plays today a very important role in constraining many extensions of the SM, and in particular the MSSM.

The experimental number most easily compared with theory is the inclusive branching ratio

$$[BR(B \rightarrow X_s\gamma)]_{exp} = (2.32 \pm 0.67) \times 10^{-4}. \quad (6)$$

In the SM, this process is described, at the partonic level ( $b \rightarrow s\gamma$ ) and at lowest order, by loop diagrams with internal top and  $W^\pm$  lines. However, the theoretical determination of the inclusive branching ratio suffers from large uncertainties, mainly due to the QCD corrections, which at the moment have been calculated only at leading order<sup>21</sup>. A conservative estimate<sup>22</sup> gives a total theoretical error of roughly 50%:

$$[BR(B \rightarrow X_s\gamma)]_{th}^{SM} = (2.55 \pm 1.28) \times 10^{-4}, \quad (7)$$

whilst other less conservative estimates give theoretical errors as low as 30%. The excellent agreement between the two determinations (6) and (7) can be taken as another piece of evidence for SM radiative corrections. This is not yet at the level of a precision test, but already represents an important constraint on possible new physics at the electroweak scale. For example, in the MSSM there are additional diagrams, corresponding to  $(t, H^\pm)$  and  $(\tilde{t}, \tilde{\chi}^\pm)$  exchange, which can give quite large contributions to the rate<sup>23</sup>. For heavy supersymmetric particles, the data disfavour a light charged Higgs. More generally, a correlation is enforced between a light charged Higgs and light stops and charginos, since one needs the right amount of negative interference to fit the data. The situation is illustrated in figure 3, which displays<sup>17</sup> contour lines of  $R_\gamma \equiv BR(B \rightarrow X_s\gamma)_{MSSM}/BR(B \rightarrow X_s\gamma)_{SM}$ , in the plane characterized by a common mass for the lightest stop and charginos (taken here to be  $\tilde{t}_R$  and  $\tilde{H}^\pm$ ) and by the charged Higgs mass, for the representative value  $\tan\beta = 1.5$ . The calculation of the next-to-leading QCD corrections, announced in a parallel session<sup>18</sup>, would allow a significant reduction of the theoretical error, and thus greatly enhance the constraints on the MSSM and on generic two-Higgs-doublet models.

## 2.3 ‘Relaxed’ MSSM

Some of the assumptions defining the MSSM are plausible but not really compulsory, even if they may find a

justification in some theoretical constructions going beyond the MSSM. When discussing the phenomenology of low-energy supersymmetry, it is important to keep an open mind and to study what happens when some of these assumptions are relaxed.

Two possibilities were discussed in the parallel sessions. The first one<sup>24</sup> consists in writing down the most general renormalizable superpotential compatible with supersymmetry and the SM gauge symmetry, which contains, besides the familiar MSSM terms of eq. (2), the additional terms

$$\Delta w = \lambda Q D^c L + \lambda' L E^c L + \lambda'' U^c D^c D^c, \quad (8)$$

where  $\lambda$ ,  $\lambda'$  and  $\lambda''$  have to be interpreted as three-index tensors in generation space. Novel analyses of the phenomenological constraints on the  $R$ -parity violating couplings of eq. (8) were discussed in the parallel sessions<sup>25,26</sup>. New bounds from non-leptonic B-decays<sup>25</sup>, Z physics at LEP<sup>26</sup> and D-decays<sup>26</sup> were presented. It was also observed<sup>25</sup> that, if enough third-generation fields are involved, some baryon- and lepton-number violating terms in (8) could coexist with couplings of order  $10^{-1}$ – $10^{-2}$ .

The second possibility consists in allowing non-universal soft supersymmetry-breaking terms. This hypothesis is subject to very stringent constraints from FCNC, as discussed in the parallel sessions<sup>27</sup>. An example is the decay  $\mu \rightarrow e\gamma$ , subject to the strong experimental bound  $BR(\mu \rightarrow e\gamma) < 5 \times 10^{-11}$ . Off-diagonal slepton mass terms in generation space, denoted here with the generic symbol  $\delta m^2$ , would contribute to the above decay at the one-loop level, and the previous limit roughly translates into  $\delta m^2/m_{\tilde{l}}^2 < 10^{-3}$ – $10^{-5}$ , if one assumes gaugino masses of the order of the average slepton mass  $m_{\tilde{l}}$  (a quite complicated parametrization is needed to formulate the bound more precisely). Similar constraints can be obtained by looking at the  $K^0$ – $\bar{K}^0$ ,  $B^0$ – $\bar{B}^0$  systems and at other flavour-changing phenomena. It is important to recall that all these bounds are naturally respected by the strict MSSM, where the only non-universality in the squark and slepton mass terms is the one induced by the renormalization group evolution from the cut-off scale  $\Lambda$  to the electroweak scale. However, the same bounds represent quite non-trivial requirements on extensions of the MSSM, such as supersymmetric grand-unified theories (SUSY GUTs) and string effective supergravities, since in general one expects non-universal contributions to the soft supersymmetry-breaking masses. Various mechanisms that could enforce the desired amount of universality, or a sufficient suppression of FCNC via approximate alignments of the fermion and sfermion mass matrices, have been presented in the mini-review by Savoy<sup>27</sup>. Another interesting recent development is the attempt<sup>28</sup> to establish a link, in the framework of SUSY GUTs, between

the magnitude of the top quark mass and the amount of FCNC expected in the resulting, ‘relaxed’ version of the MSSM. In order to do so, one defines a SUSY GUT, with universal soft mass terms, near the scale  $M_P$ , and follows the logarithmic renormalization group evolution of the model parameters from  $M_P$  to  $M_U$ : the large top Yukawa coupling controls the amount of non-universality generated at  $M_U$ . A possible limit to the predictivity of this analysis is, in my opinion, the assumption that the logarithmic RG evolution in the  $(M_U, M_P)$  interval, with  $\beta$ -functions as computed in the specific SUSY GUT model, is a good approximation. However, this criticism does not spoil the interest of such an analysis: one can introduce a general parametrization for the universality violations at  $M_U$ , or, equivalently, at the electroweak scale, and study the bounds on these parameters coming from FCNC processes; these will have to be respected by any fundamental theory that claims to predict the soft mass parameters of the MSSM.

#### 2.4 How could the MSSM be falsified?

A legitimate question, often asked when searches for new particles<sup>29</sup> are described, is the following: How could the MSSM be falsified, in the absence of new experimental discoveries?

Apart from the generic ‘naturalness’ argument, requiring the masses of supersymmetric particles to be of the order of the Fermi scale, namely smaller than a few TeV, it is difficult to establish firmer theoretical upper bounds. Attempts to quantify an acceptable ‘measure of fine-tuning’ and use it to bound from above the supersymmetric particle masses<sup>30</sup> are parametrization-dependent, and should be taken just as indications, since they do not have a solid theoretical foundation.

However, the Higgs sector of the MSSM is very tightly constrained. At the classical level, the mass of the lightest CP-even neutral Higgs boson obeys the celebrated inequality  $m_h < m_Z |\cos 2\beta|$ . This bound is shifted by the radiative corrections<sup>31</sup>. For example, the leading one-loop correction, due to the exchange of the top quark and of its scalar partners, involves a shift in the ‘22’ diagonal entry of the CP-even mass matrix,

$$(\Delta m^2)_{22} = \frac{3g^2}{8\pi^2} \frac{m_t^4}{m_W^2 \sin^2 \beta} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \dots, \quad (9)$$

which clearly exhibits the relevant dependences on the top and stop masses. Further refinements in the calculation of the radiative corrections to the MSSM Higgs masses, and in particular of the upper bound  $m_h^{\max}$  on  $m_h$ , include the parametrization of mixing effects in the stop sector (which can give in some cases an extra positive shift in  $m_h$ ), the resummation of the leading logarithms via the renormalization group (which in general decreases the upper bound on  $m_h$ ), the momentum-

dependence of the self-energies and loops of other MSSM particles (which give in general small effects). The results of a state-of-the-art calculation<sup>5</sup> are illustrated in figure 4, which displays contours of  $m_h^{\max}$  in the  $(m_t, \tan\beta)$  plane, for large average stop mass ( $m_{\text{sq}} \equiv \sqrt{(m_{t_1}^2 + m_{t_2}^2)/2}$ ) and negligible or maximal mixing effects, respectively. It should be stressed that  $m_h^{\max}$  is the maximum possible value of  $m_h$ , essentially saturated for  $m_A = 1$  TeV, but not necessarily the theoretically most probable value, since it is obtained by pushing the MSSM parameters to the limits of their plausible range of variation.

Similarly, only slightly weaker bounds can be established within supersymmetric models with non-minimal Higgs sectors<sup>5</sup>. Therefore, excluding the predicted Higgs sectors stands out as the most promising option for falsifying the MSSM and its non-minimal variants at future accelerators<sup>32</sup>. Positive evidence for supersymmetry, however, can only come from the discovery of some (R-odd) supersymmetric particle.

### 2.5 ‘Constrained’ MSSM

A remarkable fact, extensively advertised in the last few years, is the following: combining the extracted values of the effective gauge couplings at the weak scale and the leading logarithmic evolution of the latter<sup>33</sup> in the MSSM (with no new thresholds), one gets a consistent picture of approximate unification of the gauge couplings at a scale  $M_U \sim 2 \times 10^{16}$  GeV.

This stunning success, however, does not allow us to single out a unique SUSY GUT replacing the MSSM at the scale  $M_U$ ! In constructing such a theory, there is freedom to choose the unified gauge group, the representations in the Higgs sector, the parameters of the superpotential couplings (including, in general, a number of explicit mass terms), the structure of the soft terms after spontaneous supersymmetry breaking. Even choosing the simplest and most famous SUSY GUT, minimal SUSY SU(5)<sup>34</sup>, predictivity is limited by the freedom to choose the masses of some of the heavy Higgs multiplets, and by the likely existence of corrections to the SUSY-GUT Lagrangian, in the form of little-suppressed non-renormalizable operators, induced by physics at possible nearby scales (compactification scale, string scale, Planck scale). Moreover, minimal SUSY SU(5) must certainly be modified to incorporate a realistic fermion mass spectrum and to solve the doublet–triplet splitting problem.

The moral of the story is that, when performing phenomenological analyses, it may be dangerous to put bounds on the MSSM mass spectrum by imposing additional constraints such as ‘strict’ gauge coupling unification, ‘strict’ bottom–tau Yukawa coupling unification, proton decay as described by minimal SUSY SU(5), or radiative electroweak symmetry breaking with universal

soft scalar masses at  $M_U$ . Many strong (and indeed unnecessary) model dependences are introduced! Before going to this level of detail, one would need a believable theory at the scale  $M_U$  and, in my opinion, we have not yet reached such a stage. Therefore, some of the interesting analyses presented in the parallel sessions<sup>19,35,36</sup>, technically correct within their assumptions, must be interpreted with a grain of salt!

## 3 Extensions near the Planck scale (superstrings and their possible low-energy implications)

In the search for a more fundamental theory going beyond the MSSM, and allowing us to predict some of its many parameters, we have today a great advantage with respect to the early eighties, since we can make use of the impressive progress of string theories over the last decade.

Superstrings<sup>37</sup> (perhaps to be replaced, some day, by the conjectured ‘M-theory’, of which the various string theories may be different perturbative expansions) are the only known candidate for a consistent, ultraviolet-finite quantum theory of gravity, unifying all fundamental interactions. There are perturbatively stable four-dimensional solutions of the heterotic string with nice phenomenological properties such as  $N = 1$  supersymmetry in flat four-dimensional space-time, a gauge group  $G$  containing the SM gauge group  $SU(3) \times SU(2) \times U(1)$ , three chiral families (and possibly extra stuff), and more. Incidentally, the fact that supersymmetry seems to play a very important role for the quantum stability of superstring vacua may be taken as an additional motivation to favour low-energy supersymmetry over technicolor: however, it should be kept in mind that so far superstrings have not been able to give us any definite insight about the scale of supersymmetry breaking.

The general feature to be stressed is that string theories contain one explicit mass scale, the string scale, which fixes a mass unit and acts as a physical ultraviolet cut-off. All the other physical scales ( $M_P, M_U, m_Z, \dots$ , in realistic models), and all the dimensionless couplings of the low-energy effective theory (probably some version of the MSSM), are controlled by the VEVs of some scalar fields, called **moduli**, corresponding, in the effective supergravity theories, to perturbatively flat directions of the scalar potential. The inclusion of non-perturbative quantum effects is expected to spontaneously break supersymmetry and to remove the degeneracy in the moduli space, thus selecting the correct vacuum.

The special duality properties of string theories<sup>38</sup> (some of which have their counterpart at the field-theory level, as discussed at this Conference by E. Verlinde<sup>39</sup>) can play a crucial role in controlling these phenomena. The best-known string dualities are the so-called  $T$ -dualities, of which the simplest example is the equiv-

allence between a string compactified on a circle of radius  $R$  and the same string compactified on a circle of radius  $1/R$  (in appropriate string units). These dualities are perturbative, in the sense that the duality transformations do not act on the dilaton field, whose VEV controls the coupling constant associated with the string loop expansion, so they can be consistently defined in the weak-coupling limit. The dualities at the origin of a lot of recent excitement are however the so-called  $S$ -dualities, which interchange weak and strong coupling, and are therefore inherently non-perturbative. As explained by Verlinde, a prototype of  $S$ -duality is the well-known electric-magnetic duality of QED. In supersymmetric theories, electric-magnetic duality is expected to be part of a larger set of transformations, acting both on the gauge coupling  $g$ , controlling the  $F^2$  term in the Lagrangian, and on the vacuum angle  $\theta$ , controlling the associated  $F\tilde{F}$  term, combined into a single chiral superfield  $S$ . There is mounting evidence that  $S$ -duality is indeed a symmetry of the ten-dimensional heterotic string compactified on a six-torus, as well as of globally supersymmetric  $N = 4$  Yang-Mills theories. Even more interestingly, examples are being found of dual pairs of string theories, in which one string theory at strong coupling is equivalent to another string theory at weak coupling. Most of the evidence collected so far concerns string theories that would have unbroken  $N > 1$  supersymmetry in  $d = 4$ , but the physically most important goal is clearly to understand the theories with  $N = 1$  and  $N = 0$  supersymmetries in four dimensions: it would be great if one could study non-perturbative phenomena in realistic string models just by going to the dual, weakly-coupled theory! Important conceptual developments are rapidly taking place also in this respect. Waiting for solid results, applicable to realistic cases, we are already witnessing a change of perspective in the approach to some phenomenological problems. In the rest of this talk, I would like to mention some of them, not because they are particularly important, but because they are the ones in which I have recently been involved.

### 3.1 Supersymmetry breaking

At the level of dimensionless couplings, the MSSM is more predictive than the SM, since its quartic scalar couplings are related by supersymmetry to the gauge and the Yukawa couplings. The large amount of arbitrariness in the MSSM phenomenology is strictly related to its explicit mass parameters, the soft supersymmetry-breaking masses and the superpotential Higgs mass. Such arbitrariness cannot be removed within theories with softly broken global supersymmetry, such as SUSY GUTs: to make progress, *spontaneous* supersymmetry breaking must be introduced.

To discuss spontaneous supersymmetry breaking in

a realistic and consistent framework, gravitational interactions cannot be neglected. One is then led to  $N = 1$ ,  $d = 4$  supergravity, seen as an effective theory below the Planck scale, within which tree-level calculations can be performed and some qualitative features of the ultraviolet-divergent one-loop quantum corrections be studied. Of course, infrared renormalization effects can be studied, but they are plagued by the ambiguities due to the counterterms for the renormalizable operators. To proceed further, one must go to  $N = 1$ ,  $d = 4$  superstrings, seen as realizations of a fundamental ultraviolet-finite theory, within which quantum corrections to the low-energy effective action can be consistently taken into account, with no ambiguities due to the presence of arbitrary counterterms.

In recent years, two approaches to the problem have been followed. On the one hand, four-dimensional tree-level string solutions, in which  $N = 1$  local supersymmetry is spontaneously broken via orbifold compactifications, have been constructed<sup>40</sup>: none of the existing examples is fully realistic, however they represent a useful laboratory to perform explicit and unambiguous string calculations. On the other hand, many studies have been performed within string effective supergravity theories, assuming that supersymmetry breaking is induced by non-perturbative phenomena such as gaugino condensation<sup>41</sup>: the loss in predictivity is compensated by the possibility of a more general parametrization, including non-perturbative effects that are still hard to handle at the string theory level. With the advent of string-string dualities, it is even conceivable that the two approaches may be related (in an interesting paper that appeared after this Conference<sup>42</sup>, it is argued that string tree-level breaking in a type II string solution may be dual to non-perturbative breaking in a heterotic counterpart).

Before proceeding with the discussion, it may be useful to recall some basic facts of  $N = 1$ ,  $d = 4$  supergravity<sup>43</sup>. The theory can be formulated with three types of supermultiplets: in addition to the chiral and vector supermultiplets, already present in global supersymmetry, we need to introduce the gravitational supermultiplet, whose physical degrees of freedom are the spin-2 graviton and its supersymmetric partner, the spin-3/2 gravitino. Up to higher-derivative terms, the theory is completely determined by two functions of the chiral superfields: one is the Kähler function  $\mathcal{G}(z, \bar{z}) = K(z, \bar{z}) + \log |w(z)|^2$ , which controls the kinetic terms and the interactions of the chiral multiplets; this function is conventionally decomposed into a Kähler potential  $K$  and a superpotential  $w$ . The other is the gauge kinetic function  $f_{ab}(z)$ , which controls the kinetic terms and the interactions of the vector supermultiplets. It is customary to work in the natural supergravity units, where all masses are expressed in units of the Planck mass, i.e.

$M_P = 1$  by convention. An important difference with global supersymmetry is that the scalar potential is no longer positive-semidefinite, but takes the form

$$V_0 = |D_a|^2 + |F_i|^2 - 3e^{\mathcal{G}}, \quad (10)$$

where the first two terms are positive-semidefinite, in analogy with the usual F- and D-term contributions of global supersymmetry, whereas the last term, associated with the auxiliary field of the gravitational supermultiplet, is negative-definite. The novel structure of the potential in supergravity theories permits the breaking of supersymmetry with vanishing vacuum energy, if the last term in eq. (10) cancels exactly the remaining ones at the minimum: the order parameter for the breaking of local supersymmetry in flat space is the gravitino mass,  $m_{3/2}^2 = e^{\mathcal{G}}$ , which fixes the scale of all supersymmetry-breaking mass splittings, and therefore of the MSSM soft mass terms in the low-energy limit.

The generic problems to be solved by a satisfactory mechanism for spontaneous supersymmetry breaking can be succinctly summarized as follows:

- **Classical vacuum energy.** The potential of  $N = 1$  supergravity does not have a definite sign and scales as  $m_{3/2}^2 M_P^2$ : already at the classical level, one must arrange for the vacuum energy to be vanishingly small with respect to its natural scale.
- **$(m_{3/2}/M_P)$  hierarchy.** In a theory where the only explicit mass scale is the reference scale  $M_P$  (or the string scale), one must find a convincing explanation of why it is  $m_{3/2} \lesssim 10^{-15} M_P$  (as required by a natural solution to the hierarchy problem), and not  $m_{3/2} \sim M_P$ .
- **Stability of the classical vacuum.** Even assuming that a classical vacuum with the above properties can be arranged, the leading quantum corrections to the effective potential of  $N = 1$  supergravity scale again as  $m_{3/2}^2 M_P^2$ , too severe a destabilization of the classical vacuum to allow for a predictive low-energy effective theory.
- **Universality of squark/slepton mass terms.** Such a condition (or alternative but equally stringent ones) is phenomenologically necessary to adequately suppress FCNC, but is not guaranteed in the presence of general field-dependent kinetic terms.

From the above list, it should already be clear that the generic properties of  $N = 1$  supergravity are not sufficient for a satisfactory supersymmetry-breaking mechanism. Indeed, no fully satisfactory mechanism exists, but interesting possibilities arise within string effective supergravities. The best results obtained so far are listed below:

- It is possible to formulate supergravity models where the classical potential is manifestly positive-semidefinite, with a continuum of minima corresponding to broken supersymmetry and vanishing vacuum energy, and the gravitino mass sliding along a flat direction<sup>44,45</sup>. A recent development is the construction of models of this type where gauge and supersymmetry breaking are simultaneously realized, with goldstino components along gauge-non-singlet directions<sup>46</sup>.
- This special class of supergravity models emerges naturally, as a plausible low-energy approximation, from four-dimensional string models, irrespectively of the specific dynamical mechanism that triggers supersymmetry breaking. Due to the special geometrical properties of string effective supergravities, the coefficient of the one-loop quadratic divergences in the effective theory,  $\text{Str } \mathcal{M}^2$ , can be written as<sup>47</sup>

$$\text{Str } \mathcal{M}^2(z, \bar{z}) = 2Q m_{3/2}^2(z, \bar{z}), \quad (11)$$

where  $Q$  is a field-independent coefficient, calculable from the modular weights of the different fields belonging to the effective low-energy theory, i.e. the integer numbers specifying their transformation properties under the relevant duality. The non-trivial result is that the only field-dependence of  $\text{Str } \mathcal{M}^2$  occurs via the gravitino mass. Since all supersymmetry-breaking mass splittings, including those of the massive string states not contained in the effective theory, are proportional to the gravitino mass, this sets the stage for a natural cancellation of the  $\mathcal{O}(m_{3/2}^2 M_P^2)$  one-loop contributions to the vacuum energy. Indeed, there are explicit string examples that exhibit this feature. If this property can persist at higher loops (an assumption so far), then the hierarchy  $m_{3/2} \ll M_P$  can be induced by the logarithmic corrections due to light-particle loops<sup>45</sup>.

- In this special class of supergravity models one naturally obtains, in the low-energy limit where only renormalizable interactions are kept, very simple mass terms for the MSSM states ( $m_0, m_{1/2}, \mu, A, B$  in the standard notation), calculable via simple algebraic formulae from the modular weights of the corresponding fields and easily reconcilable with the phenomenological universality requirements<sup>47</sup>. This last result can indeed be obtained also in a slightly less restrictive framework<sup>48</sup>.

Just to give the flavour of the argument, we present here an ultra-simplified example, which retains the relevant qualitative features of the general case, without its full technical complexity.

Consider a supergravity theory containing as chiral superfields a gauge-singlet  $T$  (to be thought of as one of



the superstring moduli fields), and a number of charged fields  $C^\alpha$  (to be thought of as the matter fields of the MSSM and possibly others), with Kähler potential

$$K = -3\log(T + \bar{T}) + \sum_\alpha |C^\alpha|^2 (T + \bar{T})^{\lambda_\alpha} + \dots, \quad (12)$$

and superpotential

$$w_{SUSY} = d_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma. \quad (13)$$

The model exhibits a classical invariance under the following set of transformations, parametrizing the continuous group  $SL(2, R)$ :

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad C^\alpha \rightarrow (icT + d)^{\lambda_\alpha} C^\alpha, \quad (ab - cd = 1). \quad (14)$$

The above symmetry can be interpreted as an approximate low-energy remnant of a  $T$ -duality invariance under the discrete group  $SL(2, Z)$ , corresponding to the restriction of the transformations (14) to the case of integer  $(a, b, c, d)$  coefficients, and generated by the two transformations  $T \rightarrow 1/T$  and  $T \rightarrow T + i$ . One can think of this  $SL(2, Z)$  as an exact quantum symmetry of the underlying string model. In the language of supergravity, the Kähler potential transforms as  $K \rightarrow K + \phi + \bar{\phi}$ , where  $\phi$  is an analytic function, and the superpotential as  $w \rightarrow w \exp(-\phi)$ , so that the full Kähler function  $\mathcal{G}$  remains invariant.

Without specifying the dynamics which induces the spontaneous breaking of local supersymmetry, one can try to parametrize the latter with a superpotential modification of the form

$$w = w_{SUSY} + \Delta w, \quad \Delta w = k \neq 0, \quad (15)$$

where  $k$  is a constant, independent of the modulus field  $T$ , which can be thought of as the large- $T$  limit of a modular form of  $SL(2, Z)$ . In the case in which other moduli fields are present, such as the dilaton-axion field  $S$  associated with the gauge coupling constant, one can replace  $k$  with a suitable function of  $S$ , with the correct transformation properties under a possible  $S$ -duality. Notice that the superpotential modification introduced above breaks the invariance under  $T \rightarrow 1/T$ , but preserves the shift symmetry  $T \rightarrow T + i\alpha$ . A low-energy structure equivalent to the one introduced here has been found in explicit constructions of string orbifold models with string tree-level breaking<sup>40</sup>, but these results could have more general validity, and apply also, with the appropriate modifications, to the case of non-perturbative breaking.

In the supergravity theory defined above, by applying the standard formalism one can easily verify the following results:

- Thanks to the identity  $|F_T|^2 \equiv 3e^{\mathcal{G}}$ , the scalar potential of eq. (10) is automatically positive-semidefinite. At any minimum of the potential

supersymmetry is broken and the gravitino mass,  $m_{3/2}^2 = k^2/(T + \bar{T})^3 \neq 0$  if one takes for simplicity  $C^\alpha = 0$ , is classically undetermined. The modulus field  $T$  corresponds to a flat direction, as in the no-scale models<sup>44</sup>, and its fermionic partner  $\tilde{T}$  plays the role of the goldstino in the super-Higgs mechanism.

- Str  $\mathcal{M}^2$  can be put in the form of eq. (11), with

$$Q = -2 + \sum_\alpha (1 + \lambda_\alpha), \quad (16)$$

where the first addendum is the contribution of the massive gravitino and the second one the contribution of the matter fields.

- In MSSM notation, the following very simple mass terms are generated:

$$\frac{(m_0^2)_\alpha}{m_{3/2}^2} = 1 + \lambda_\alpha, \quad (17)$$

$$\frac{(A)_{\alpha\beta\gamma}}{m_{3/2}} = 3 + \lambda_\alpha + \lambda_\beta + \lambda_\gamma, \quad (18)$$

$$\frac{(\mu)_{\alpha\beta}}{m_{3/2}} = 1 + \frac{\lambda_\alpha + \lambda_\beta}{2}, \quad (19)$$

$$\frac{(B)_{\alpha\beta}}{m_{3/2}} = 2 + \frac{\lambda_\alpha + \lambda_\beta}{2}. \quad (20)$$

The above example can be easily generalized to include gauge interactions, with a non-trivial moduli dependence of the gauge kinetic function: non-vanishing gaugino masses can then be generated, proportional to the gravitino mass, and eq. (16) can be modified accordingly. It is important to stress that, in this framework, the phenomenologically desirable universality properties of the soft mass terms can naturally arise as a consequence of  $T$ -duality. Furthermore, a non-vanishing  $\mu$ -term can be generated for the MSSM, proportional to  $m_{3/2}$ , even if the supergravity superpotential does not contain any explicit Higgs mass term.

The weakest point of the above construction is the absence of a string calculation showing that, if there is cancellation of the  $\mathcal{O}(m_{3/2}^2 M_P^2)$  contributions to the effective potential at one loop, this cancellation can persist at higher loops. Since in the effective theory one can identify some quadratically divergent two-loop graphs<sup>49</sup>, such an assumption is far from obvious. However, there are hints<sup>47</sup> that the numerical coefficient of eq. (16) might be given a topological interpretation, so such an assumption is not completely arbitrary.

Under the assumption that no terms  $\mathcal{O}(m_{3/2}^2 M_P^2)$  are generated by string quantum corrections to the effective potential, the possibility arises of treating the gravitino mass  $m_{3/2}$  as a dynamical variable of the low-energy theory valid near the electroweak scale, namely

the MSSM. Then the actual magnitude of the gravitino mass could be determined by the logarithmic quantum corrections<sup>45</sup>, as computed in the MSSM. The minimization condition of the one-loop effective potential  $V_1$ , with respect to  $m_{3/2}$ , would take the form<sup>50</sup>:

$$m_{3/2}^2 \frac{\partial V_1}{\partial m_{3/2}^2} = 2V_1 + \frac{\text{Str } \mathcal{M}^4}{64\pi^2} = 0. \quad (21)$$

The above equation can be interpreted as defining an infrared fixed point for the vacuum energy, with the two terms in the second member representing the canonical scaling and the scaling violation by quantum corrections, respectively. One can show that, for reasonable values of the boundary conditions on the dimensionless parameters, an exponentially suppressed hierarchy  $m_{3/2} \ll M_P$  can be generated.

Of course, the reason why  $m_{3/2}$  can be treated as a dynamical variable in the effective low-energy theory is the existence of a very flat direction for the modulus on which it depends monotonically. This means that, after the inclusion of the  $\mathcal{O}(m_{3/2}^4)$  quantum corrections, there will be some very light gauge-singlet spin-0 fields, with ‘axion-like’ or ‘dilaton-like’ couplings and masses  $\mathcal{O}(m_{3/2}^2/M_P)$ , i.e. in the  $10^{-3}$ – $10^{-4}$  eV range if  $m_{3/2}^2 \sim G_F^{-1}$ , with interesting astrophysical and cosmological implications, including a number of potential phenomenological problems<sup>51</sup>.

### 3.2 Infrared moduli physics and the flavour problem

Once the taboo has been broken, by considering a parameter of the MSSM (in the previous example, the overall scale of its mass terms) as a dynamical variable at the electroweak scale, and some partial success obtained (a possible explanation for the  $m_{3/2} \ll M_P$  hierarchy, at the price of one important assumption and some unsolved cosmological problems), it is not a big step to generalize the game to other MSSM parameters, to see if there is a chance that other problems can be solved.

For example, in the case of non-universal soft mass terms one has in general a very severe problem with FCNC<sup>27</sup>, unless one can find a good reason to justify the alignment of the quark and squark mass matrices. It has recently been proposed<sup>52</sup> that also the relative angles between the quark and squark mass matrices may be considered as dynamical variables: again, minimization of the vacuum energy can induce at least partial alignment. Analogous considerations have also been made<sup>53</sup> by considering a possible dynamics just above the SUSY-GUT scale  $M_U$ .

Along similar lines, two groups<sup>50,54,55</sup> have considered the possibility of treating some of the Yukawa couplings of the MSSM as dynamical variables, as reviewed in a parallel session<sup>56</sup>. The goal is to find a dynamical ex-

planation for the numerical values of some of the fermion masses, for example those of the third-generation quarks.

To review the logic of the argument, we begin by recalling that, in the MSSM, the RGEs for the top and bottom Yukawa couplings admit an effective infrared fixed curve, analogous to the effective infrared fixed point that one obtains<sup>57</sup> by setting the bottom Yukawa coupling to zero. Neglecting for simplicity the  $\tau$  Yukawa coupling and the electroweak gauge couplings, an approximate analytical equation for the infrared fixed curve is<sup>58,55</sup>

$$\alpha_t + \alpha_b \lesssim \frac{2\pi E}{3F} \cdot f \left[ \frac{4\alpha_t\alpha_b}{(\alpha_t + \alpha_b)^2} \right], \quad (22)$$

where to a good approximation  $(2\pi E/3F) \simeq (8/9)\alpha_S$  and  $f$  is a hypergeometric function bounded by  $1 \leq f \leq 12/7$ . This infrared behaviour is illustrated<sup>55</sup> in figure 5. The previous considerations are only sufficient to set an upper bound, of order 200 GeV, on the combination  $m_t^2/\sin^2\beta + m_b^2/\cos^2\beta$ , but cannot predict the values of the top and bottom quark masses.

However, if after supersymmetry breaking some very flat directions are left over in moduli space, and the Yukawa couplings have some functional dependence on the corresponding fields, then also the top and bottom Yukawa couplings can be treated as dynamical variables of the low-energy theory. A point stressed in a parallel session is that, in this two-variable problem, the details of the moduli-dependence of the two Yukawa couplings may or may not impose some constraints on the minimization problem to be solved. Irrespectively of these details, it can be shown that minimization of the vacuum energy almost invariably brings the low-energy couplings very close to the infrared fixed curve of fig. 5b. Unconstrained minimization, however, favours the solution  $h_t = h_t^{max} \gg h_b = 0$ . Constrained minimization, instead, can produce the phenomenologically desired solution  $h_t \simeq h_t^{max} \gg h_b \neq 0$ , provided that some constraint in the moduli space forbids the configuration  $h_b(M_U) = 0$ . Barring these model-dependent details, which could be worked out only in a specific string construction, one is still left with a definite prediction

$$(M_t^{IR})^2 \lesssim \frac{m_t^2}{\sin^2\beta} + \frac{m_b^2}{\cos^2\beta} \lesssim \frac{12}{7}(M_t^{IR})^2, \quad (23)$$

where  $M_t^{IR} \simeq (4/3)\sqrt{\alpha_3/(\alpha_2 + \alpha')}m_Z \simeq 195$  GeV. It seems difficult to go beyond the above result without a more detailed knowledge of the moduli-dependence of Yukawa couplings in string theories and of the mechanism for spontaneous supersymmetry breaking.

## 4 Conclusions

Among the SM extensions at the Fermi scale, the MSSM stands out as the theoretically most motivated and the

phenomenologically most viable one. In view of its limitations, as far as uniqueness and predictivity are concerned, the MSSM should be taken as a useful phenomenological parametrization, with the hope that its many parameters will be fixed in the not-too-distant future by the discovery of supersymmetric particles at LEP II, Tevatron and the LHC. Paraphrasing an expression used in the parallel sessions, this would be the real ‘bottom-up approach’!

On the more theoretical side, intense study over the last ten years increasingly suggests that strings must be taken seriously as a candidate fundamental theory defined near the Planck scale. The connection between string theories and the MSSM, which we would like to understand as the low-energy effective field theory near the correct string vacuum, is made difficult by the problem of spontaneous supersymmetry breaking. There is great hope that we may soon extend the exciting results on non-perturbative dualities in supersymmetric field theories and string theories to more and more realistic situations, and promising lines of development have continued to flourish in the months between the end of this Conference and the preparation of this written contribution.

To conclude the talk, I would like to express my personal belief: both in experiment and in theory, we are heading to very exciting times!

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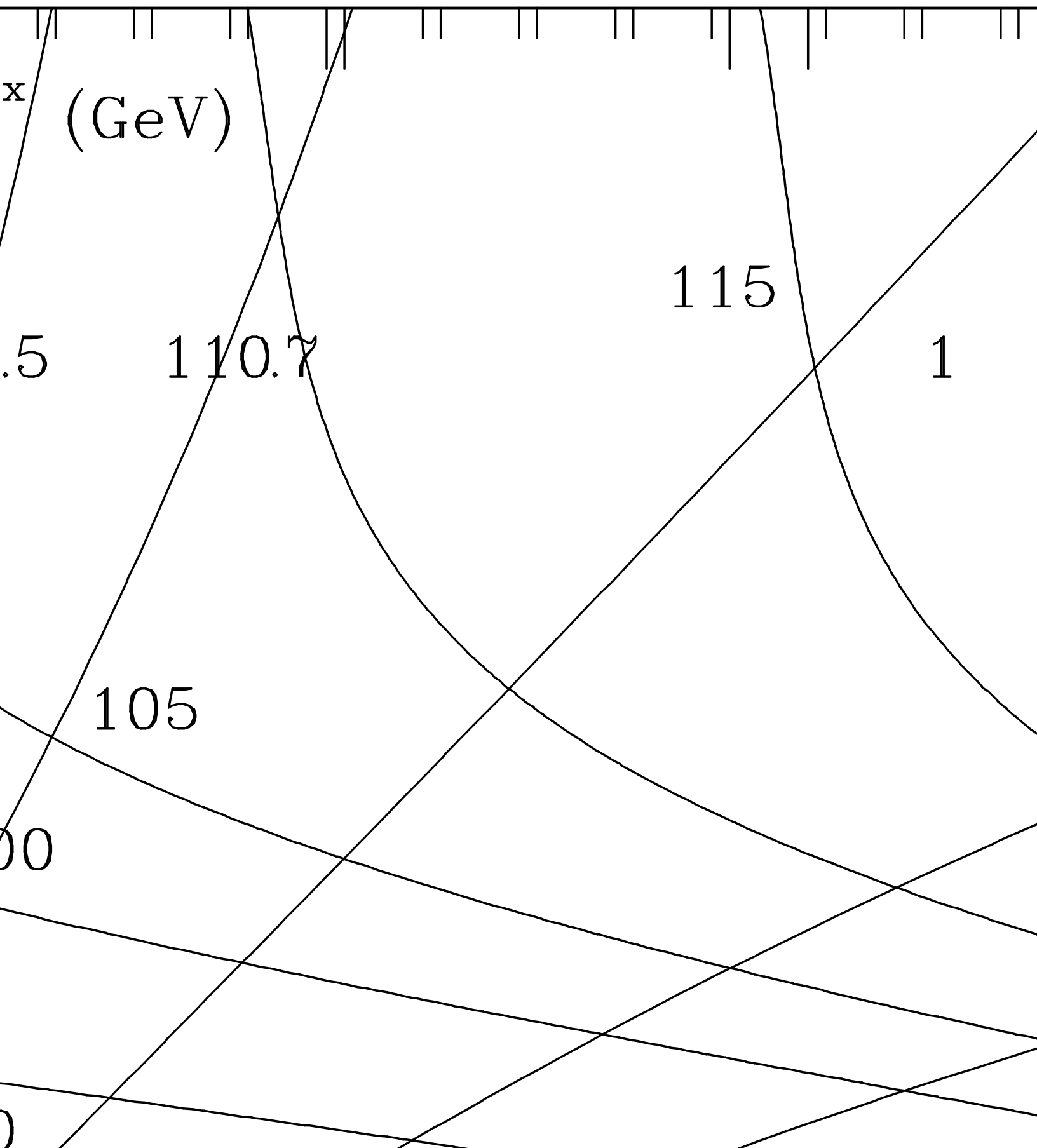
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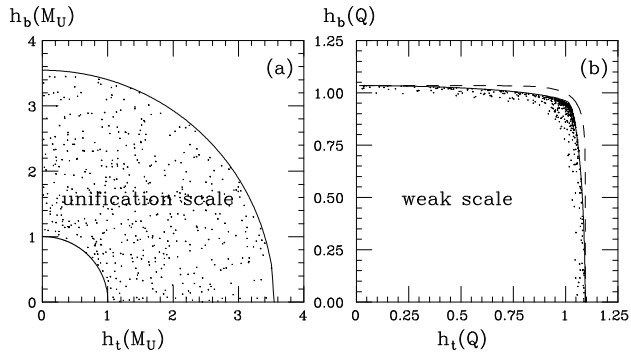


Fig. 1

Figure 5: Mapping of the  $(h_t(M_U), h_b(M_U))$  plane into the  $(h_t(Q), h_b(Q))$  plane, for  $Q = 200$  GeV. In (b), the dots correspond to the exact numerical solutions of the one-loop RGE, for the boundary conditions given in (a); the solid and dashed lines correspond to two different approximate analytical solutions.

## Extensions of the Standard Model

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### Abstract

This talk begins with a brief general introduction to the extensions of the Standard Model, reviewing the ideology of effective field theories and its practical implications. The central part deals with candidate extensions near the Fermi scale, focusing on some phenomenological aspects of the Minimal Supersymmetric Standard Model. The final part discusses some possible low-energy implications of further extensions near the Planck scale, namely superstring theories.

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